

- Lack of spatial
DOS

A quick reminder
of last week's
lecture...

- Lack of spatial dependence on the DOS
- Green function formalism for the free-electron gas

Schroedinger equation (in operator form ?):

$$-\frac{\hbar^2}{2m} \nabla^2 \psi(\vec{r}) + V(\vec{r})\psi(\vec{r}) = E\psi(\vec{r})$$

$$Eg(\vec{r}, \vec{r}') + \frac{\hbar^2}{2m} \nabla^2 g(\vec{r}, \vec{r}') - V(\vec{r})g(\vec{r}, \vec{r}') = \delta(\vec{r} - \vec{r}')$$

$$\left[E - H(\vec{r}) \right] g(\vec{r}, \vec{r}') = \delta(\vec{r} - \vec{r}')$$

$$E \langle \vec{r} | \hat{g} | \vec{r}' \rangle - \int d\vec{r}'' \langle \vec{r} | \hat{H} | \vec{r}'' \rangle \langle \vec{r}'' | \hat{g} | \vec{r}' \rangle = \langle \vec{r} | \vec{r}' \rangle$$

$$\left(E\hat{1} - \hat{H} \right) \hat{g} = \hat{1} \quad \Rightarrow \quad \boxed{\hat{g} = \left(E\hat{1} - \hat{H} \right)^{-1}}$$

$$\rho(E) = -\frac{1}{\pi} \text{Im}[g(\vec{r}, \vec{r})]$$

- Density of states (DOS) is a fundamental quantity in physics
- A number of other quantities are dependent on the DOS (e.g. number of particles, total energy, specific heat, conductivity, etc)

3D

$$g(\vec{r}, \vec{r}') = \frac{e^{i\sqrt{2mE/\hbar^2} |\vec{r} - \vec{r}'|}}{4\pi |\vec{r} - \vec{r}'|}$$

Analogously for 1D
and 2D...

2D

$$g(\vec{r}, \vec{r}') = \frac{-i}{4} H_0^{(1)}(\sqrt{2mE/\hbar^2} |\vec{r} - \vec{r}'|)$$

1D

$$g(\vec{r}, \vec{r}') = \frac{-i\hbar}{\sqrt{8mE}} e^{i\sqrt{2mE/\hbar^2} |\vec{r} - \vec{r}'|}$$

What if the particle is not free ?

Schroedinger equation in energy/frequency-domain:

$$-\frac{\hbar^2}{2m} \nabla^2 \psi(\vec{r}) + V(\vec{r})\psi(\vec{r}) = E\psi(\vec{r}) \quad \text{or} \quad H(\vec{r})\psi(\vec{r}) = E\psi(\vec{r})$$

$$H(\vec{r}) = H_0(\vec{r}) + V(\vec{r})$$

free-particle Hamiltonian

The free-particle GF satisfies

$$\{E - H_0(\vec{r})\}g(\vec{r}, \vec{r}') = \delta(\vec{r} - \vec{r}')$$

The Green function associated with H satisfies

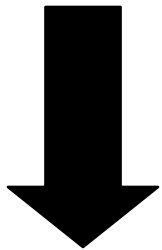
$$\{E - H(\vec{r})\}G(\vec{r}, \vec{r}') = \delta(\vec{r} - \vec{r}')$$

$$\{E - H_0(\vec{r})\}G(\vec{r}, \vec{r}') = \delta(\vec{r} - \vec{r}') + V(\vec{r})G(\vec{r}, \vec{r}')$$

The full expansion can be expressed in operator form as:

$$\hat{G} = \hat{g} + \hat{g} \hat{V} \hat{g} + \hat{g} \hat{V} \hat{g} \hat{V} \hat{g} + \hat{g} \hat{V} \hat{g} \hat{V} \hat{g} \hat{V} \hat{g} + \dots$$

$$\hat{G} = \hat{g} + \hat{g} \hat{V} \underbrace{\left\{ \hat{g} + \hat{g} \hat{V} \hat{g} \hat{V} \hat{g} + \hat{g} \hat{V} \hat{g} \hat{V} \hat{g} \hat{V} \hat{g} + \dots \right\}}_{\hat{G}}$$



Dyson's equation

$$\boxed{\hat{G} = \hat{g} + \hat{g} \hat{V} \hat{G}}$$



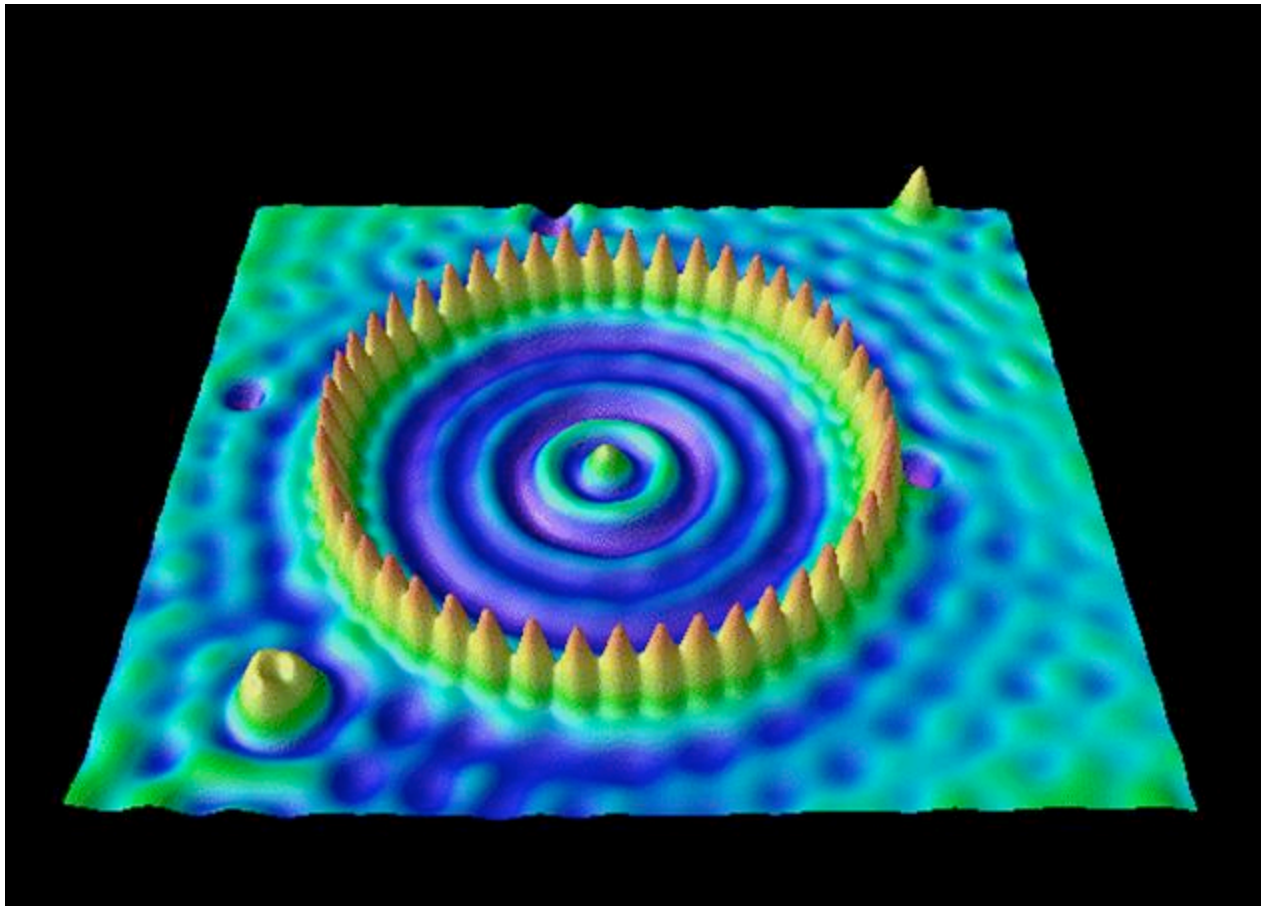
Concise way of expressing the perturbed GF in terms of the unperturbed one.

$$G(x, x) = \frac{-i}{2q} - \frac{\lambda e^{2iq|x|}}{2q(2q + i\lambda)}$$

- LDOS oscillates away from the scatterer.
- Energy-dependent oscillation periods
- Oscillation amplitude does not decay !!! (?)
- Effect of impurity needs integration over energy

You might have seen this...

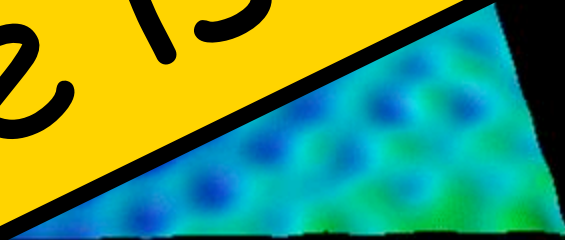
Quantum corral



You might have seen this

Quantum

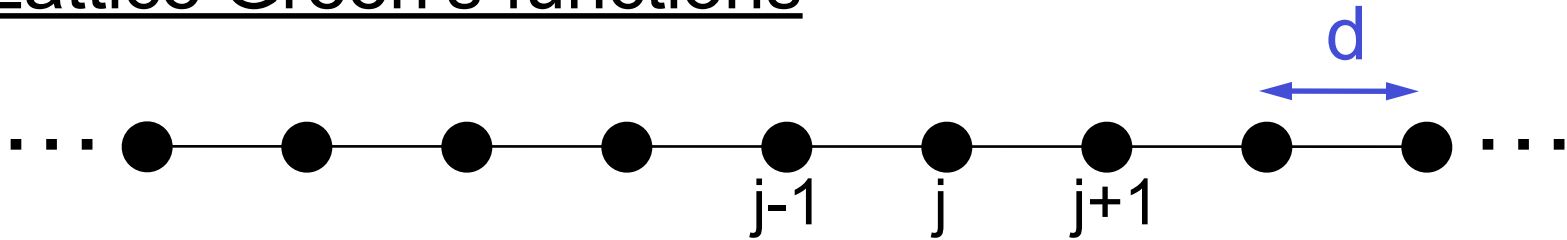
That was last
lecture. Today's
lecture is ...



OUTLINE

- Lattice Green function (back to the tight-binding model)

Lattice Green's functions



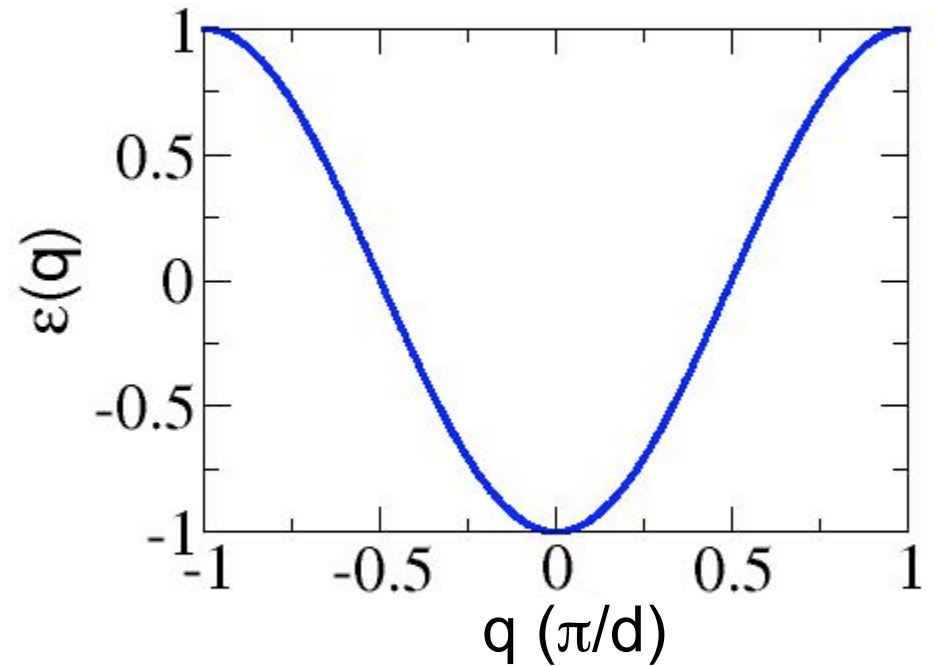
Consider the 1D **tight-binding Hamiltonian** in the basis of atomic orbitals centred at site j :

$$\hat{H} = \sum_j |j\rangle \varepsilon_0 \langle j| + \sum_j [|j\rangle t \langle j+1| + |j+1\rangle t \langle j|]$$

$$H = \begin{bmatrix} \varepsilon_0 & t & 0 & 0 \\ t & \varepsilon_0 & t & 0 \\ 0 & t & \varepsilon_0 & t \\ 0 & 0 & t & \varepsilon_0 \end{bmatrix} \longrightarrow \text{Tri-diagonal matrix}$$

We introduce a new basis

$$\begin{cases} |q\rangle = \frac{1}{\sqrt{N}} \sum_j e^{iqjd} |j\rangle \\ |j\rangle = \frac{1}{\sqrt{N}} \sum_q e^{-iqjd} |q\rangle \end{cases}$$



$$\hat{H} = \sum_{j=-\infty}^{+\infty} |j\rangle \varepsilon_0 \langle j| + \sum_{j=-\infty}^{+\infty} [|j\rangle t \langle j+1| + |j+1\rangle t \langle j|]$$

$$\hat{H} = \sum_q |q\rangle \underbrace{[\varepsilon_0 + 2t \cos(qd)]}_{\varepsilon(q)} \langle q|$$

H is diagonal



Eigenfunctions of H

$$\hat{g} = \left(E\hat{1} - \hat{H} \right)^{-1} \quad ; \quad \hat{g}(E) = \sum_q \frac{|q\rangle\langle q|}{E - \varepsilon(q)}$$

In the basis $|j\rangle \Rightarrow g = \begin{bmatrix} E - \varepsilon & -t & 0 & 0 \\ -t & E - \varepsilon & -t & 0 \\ 0 & -t & E - \varepsilon & -t \\ 0 & 0 & -t & E - \varepsilon \end{bmatrix}^{-1}$

$$\langle j | \hat{g}(E) | \ell \rangle = g_{j,\ell}(E) = \sum_q \frac{\langle j | q \rangle \langle q | \ell \rangle}{E - \varepsilon(q)}$$

$$\langle j | q \rangle = \frac{e^{iqjd}}{\sqrt{N}}$$

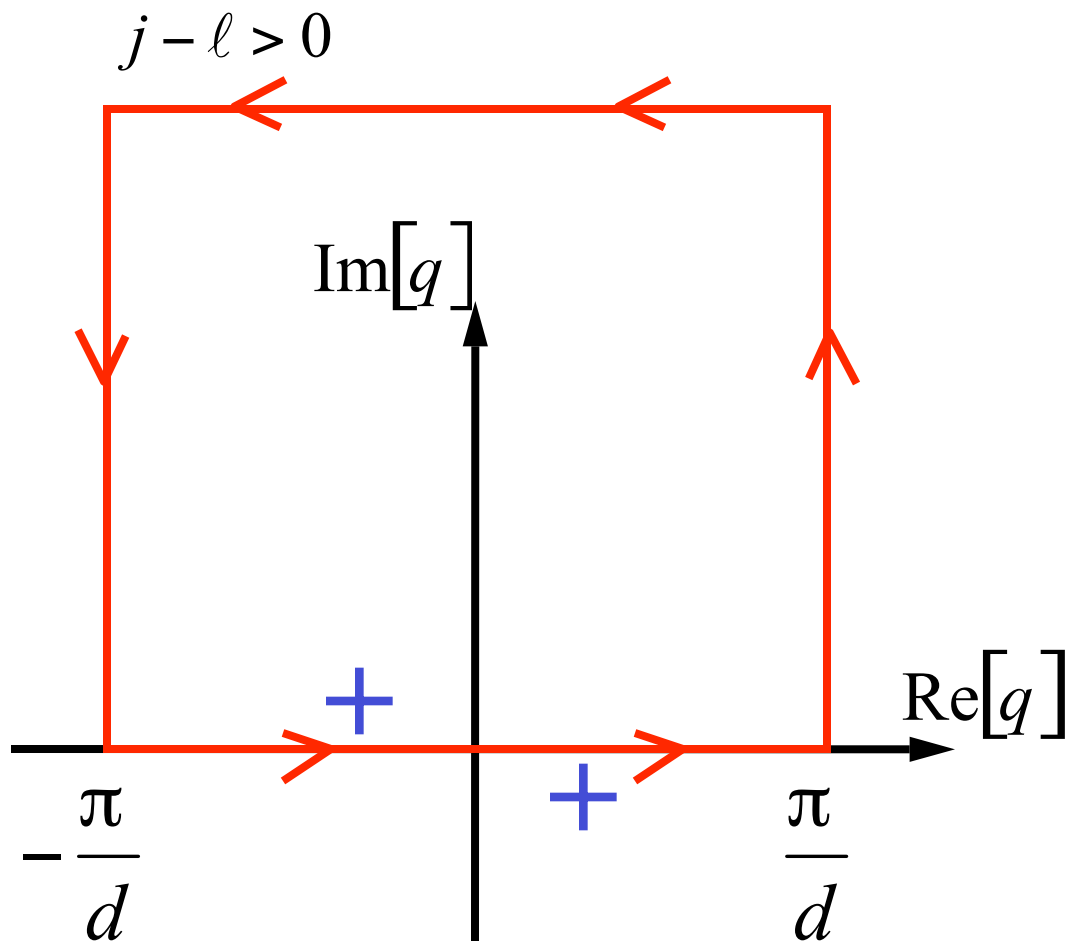
$$g_{j,\ell}(E) = \frac{1}{N} \sum_q \frac{e^{iq(j-\ell)d}}{E - \varepsilon(q)} = \frac{d}{2\pi} \int_{-\frac{\pi}{d}}^{\frac{\pi}{d}} dq \frac{e^{iq(j-\ell)d}}{E - \varepsilon(q)}$$

$$g_{j,\ell}(E) = \frac{d}{2\pi} \int_{-\frac{\pi}{d}}^{\frac{\pi}{d}} dq \frac{e^{iq(j-\ell)d}}{E - \varepsilon(q)}$$

$$g_{j,\ell}(E) = \frac{d}{2\pi} \int_{-\frac{\pi}{d}}^{\frac{\pi}{d}} dq \frac{e^{iq(j-\ell)d}}{E - \varepsilon_0 - 2t \cos(qd)}$$

$$g_{j,\ell}(E) = \frac{-d}{4\pi t} \int_{-\frac{\pi}{d}}^{\frac{\pi}{d}} dq \frac{e^{iq(j-\ell)d}}{\cos(qd) - \left(\frac{E - \varepsilon_0}{2t} \right)}$$

$$g_{j,\ell}(E) = \frac{-d}{4\pi t} \int_{-\frac{\pi}{d}}^{\frac{\pi}{d}} dq \frac{e^{iq(j-\ell)d}}{\cos(qd) - \left(\frac{E - \varepsilon_0}{2t}\right)}$$



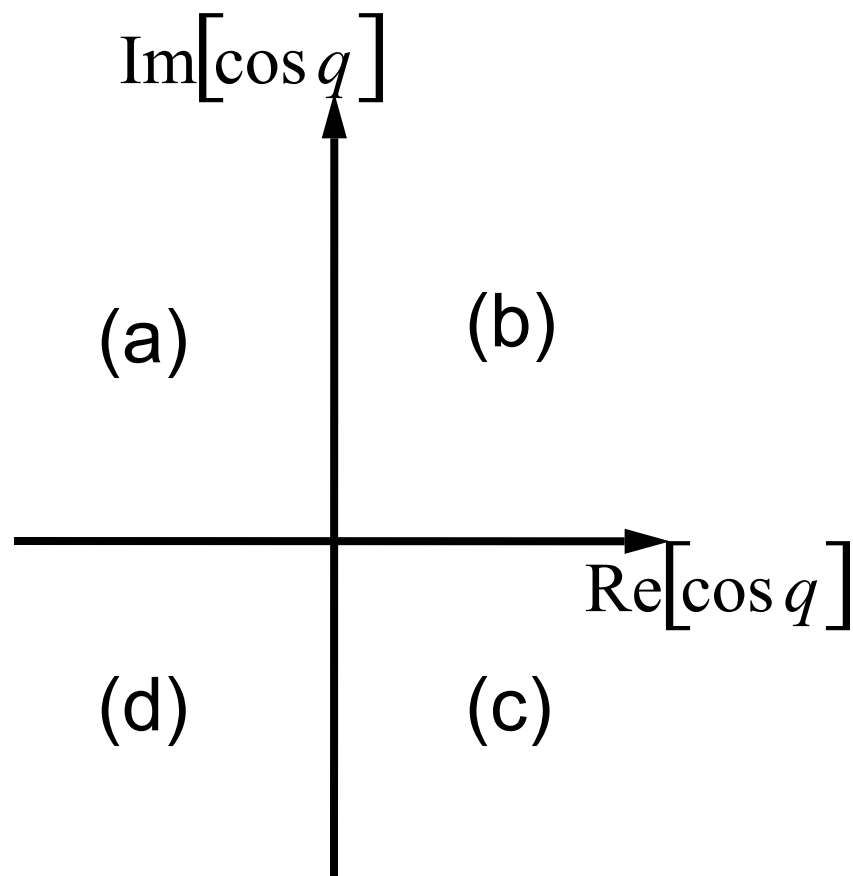
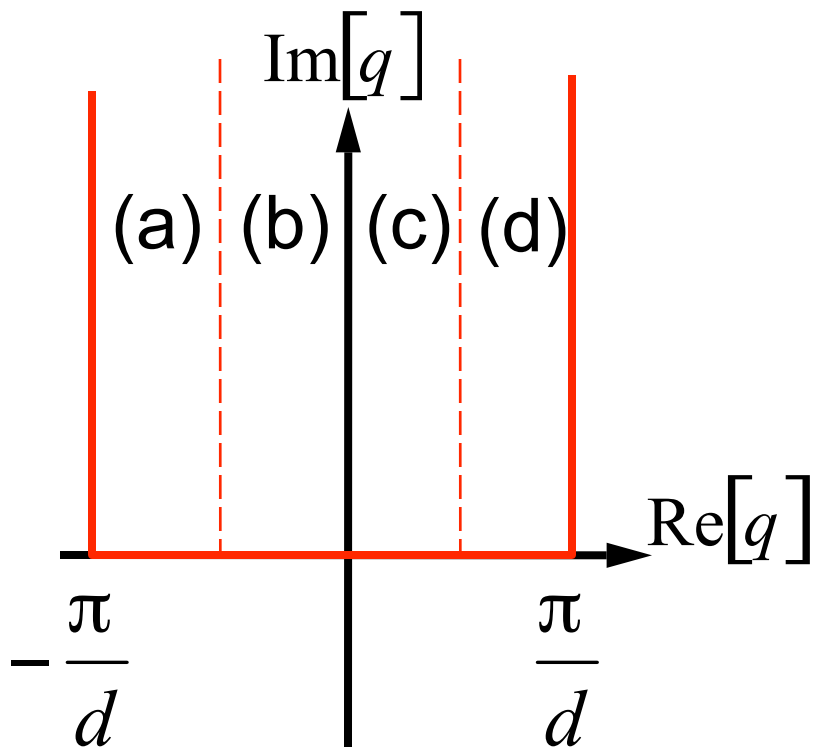
Poles:

$$\cos(q_p d) - \left(\frac{E - \varepsilon_0}{2t}\right) = 0$$

$$q_p = \frac{1}{d} \cos^{-1}\left(\frac{E - \varepsilon_0}{2t}\right)$$

$$q_p = \frac{1}{d} \cos^{-1} \left(\frac{E - \varepsilon_0}{2t} \right)$$

$$\cos(q_p d) = \left(\frac{E - \varepsilon_0}{2t} \right)$$

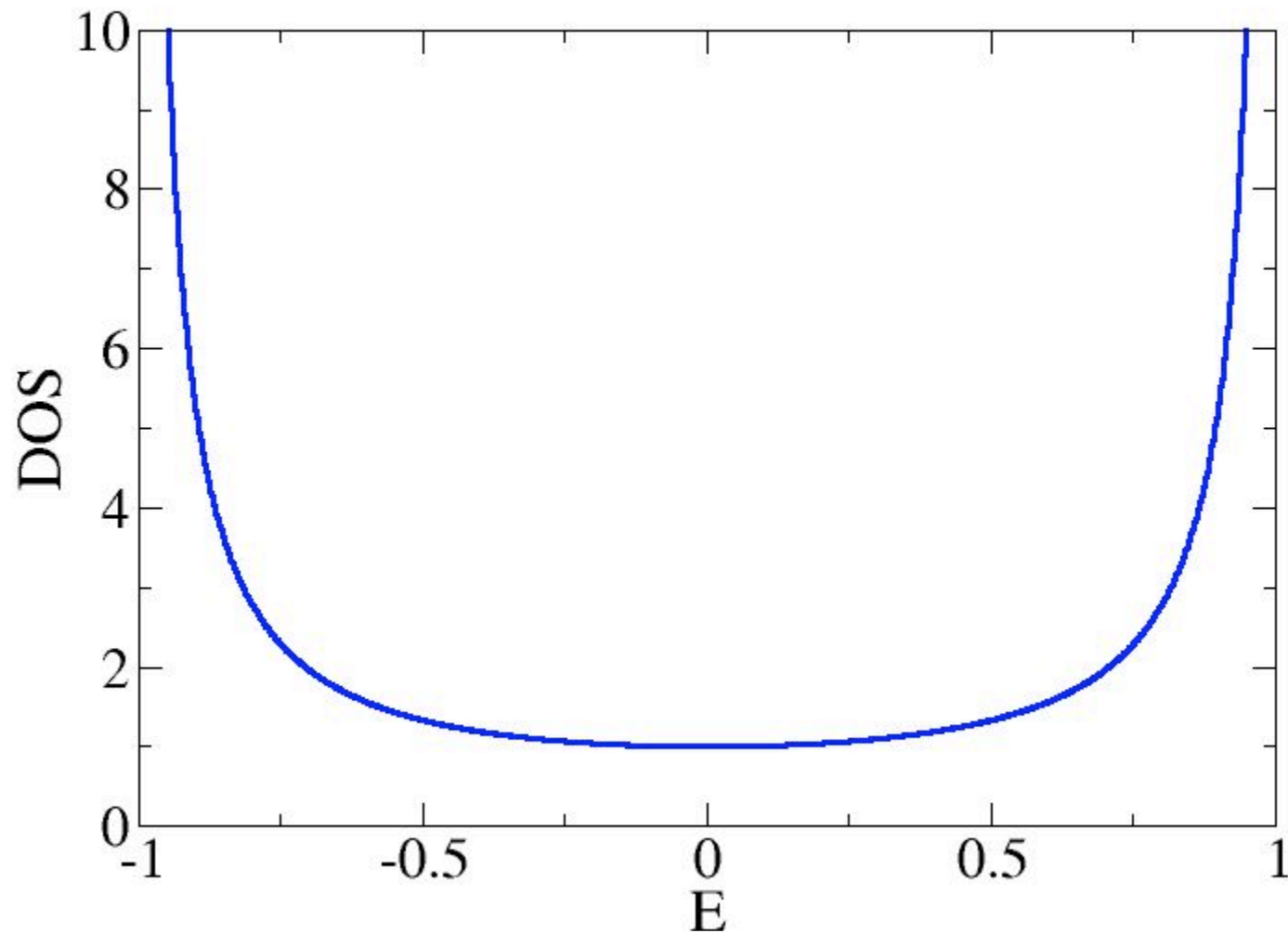


$$g_{j,\ell}(E) = \frac{-d}{4\pi t} \int_{-\frac{\pi}{d}}^{\frac{\pi}{d}} dq \frac{e^{iq(j-\ell)d}}{\cos(qd) - \left(\frac{E - \varepsilon_0}{2t}\right)}$$

$$g_{j,\ell}(E) = \left(\frac{i}{2t}\right) \frac{e^{iq_p(j-\ell)d}}{\sin(q_p d)} = \left(\frac{\pm i}{2t}\right) \frac{e^{i \cos^{-1}\left(\frac{E - \varepsilon_0}{2t}\right)(j-\ell)}}{\sqrt{1 - \left(\frac{E - \varepsilon_0}{2t}\right)^2}}$$

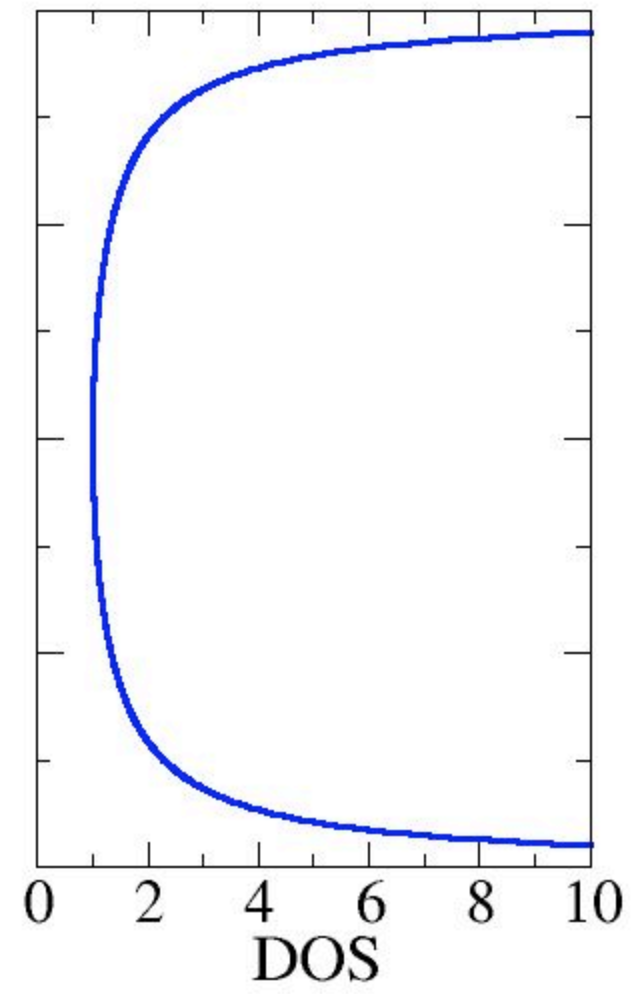
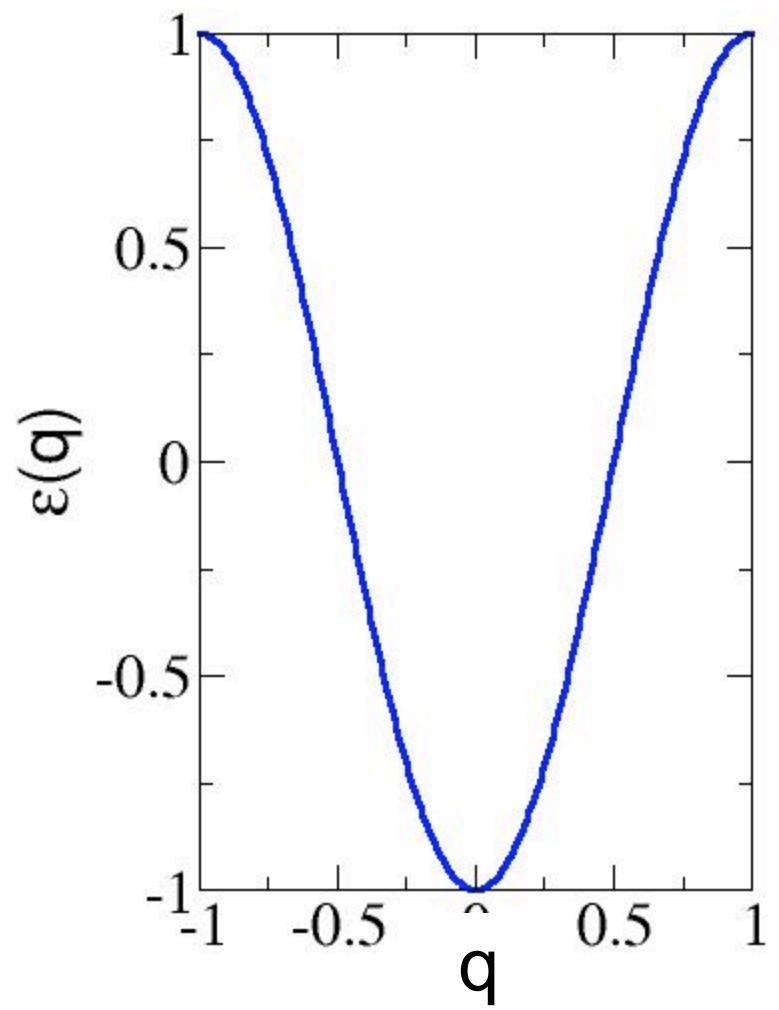
$$g_{j,\ell}(E) = \frac{\pm e^{i \cos^{-1}\left(\frac{E - \varepsilon_0}{2t}\right)(j-\ell)}}{\sqrt{(E - \varepsilon_0)^2 - 4t^2}}$$

$$\rho(E) = \frac{-1}{\pi} \text{Im} \left[g_{j,j}(E) \right] \quad g_{j,j}(E) = \frac{\pm 1}{\sqrt{(E - \varepsilon_0)^2 - 4t^2}}$$

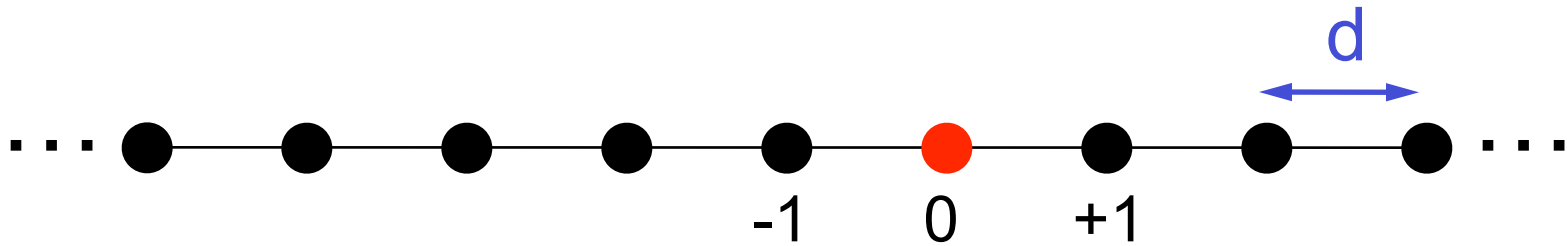


In agreement with:

$$\rho(E) \propto \int_{S_C} dS \frac{1}{|\vec{\nabla} \varepsilon(\vec{k})|}$$



Substitutional impurity



$$\hat{H} = \hat{H}_0 + \hat{V} \quad ; \quad \hat{V} = |0\rangle\lambda\langle 0|$$

$$\hat{G} = \hat{g} + \hat{g}\hat{V}\hat{G} \quad \Rightarrow \quad G_{j,l} = g_{j,l} + g_{j,0}\lambda G_{0,l} \quad ?$$

$$G_{0,l} = g_{0,l} + g_{0,0}\lambda G_{0,l} \quad \Rightarrow \quad G_{0,l} = [1 - g_{0,0}\lambda]^{-1} g_{0,l}$$

$$G_{j,l} = g_{j,l} + g_{j,0}\lambda [1 - g_{0,0}\lambda]^{-1} g_{0,l}$$

$$G_{j,l} = g_{j,l} + g_{j,0} T_{0,0} g_{0,l} \quad ; \quad T_{0,0} = \lambda [1 - g_{0,0}\lambda]^{-1}$$

$$G_{j,\ell} = g_{j,\ell} + g_{j,0} T_{0,0} g_{0,\ell} \quad ; \quad T_{0,0} = \lambda [1 - g_{0,0} \lambda]^{-1}$$

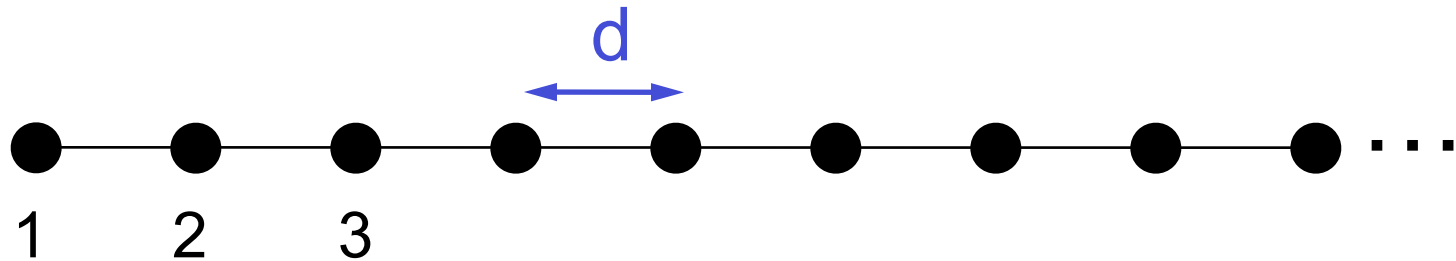
$$G_{j,j} = g_{j,j} + g_{j,0} T_{0,0} g_{0,\ell}$$

$$G_{j,j} = \frac{i}{2t \sin(q_p d)} - \frac{e^{2iq_p j d}}{4t^2 \sin^2(q_p d)} T_{0,0}$$

Analogously to the free-particle case ...

- LDOS oscillates away from the scatterer.
- Energy-dependent oscillation periods
- Oscillation amplitude does not decay !!!
- Effect of impurity needs integration over energy

Surface GF



Consider a semi-infinite linear chain. The tight-binding Hamiltonian associated with the infinite chain is:

$$\hat{H} = \sum_j |j\rangle \varepsilon_0 \langle j| + \sum_j [|j\rangle t \langle j+1| + |j+1\rangle t \langle j|]$$

$$\hat{H}|q\rangle = \underbrace{[\varepsilon_0 + 2t \cos(qd)]}_{\varepsilon(q)} |q\rangle \quad ; \quad |q\rangle = \frac{1}{\sqrt{N}} \sum_j e^{iqjd} |j\rangle$$

We introduce a linear combination of the vectors $|q\rangle$ and $|-q\rangle$:

$$|\psi_q\rangle \equiv \frac{1}{\sqrt{2}} [|q\rangle - |-q\rangle]$$

$$\hat{H}|\psi_q\rangle = \frac{1}{\sqrt{2}} [\varepsilon(q)|q\rangle - \varepsilon(-q)|-q\rangle] \quad \varepsilon(q) = \varepsilon(-q)$$

$$\hat{H}|\psi_q\rangle = \varepsilon(q)|\psi_q\rangle$$

$|\psi_q\rangle$ is an eigenfunction of H with the same eigenvalue

It is convenient to project the function $|\psi_q\rangle$ on the sites j :

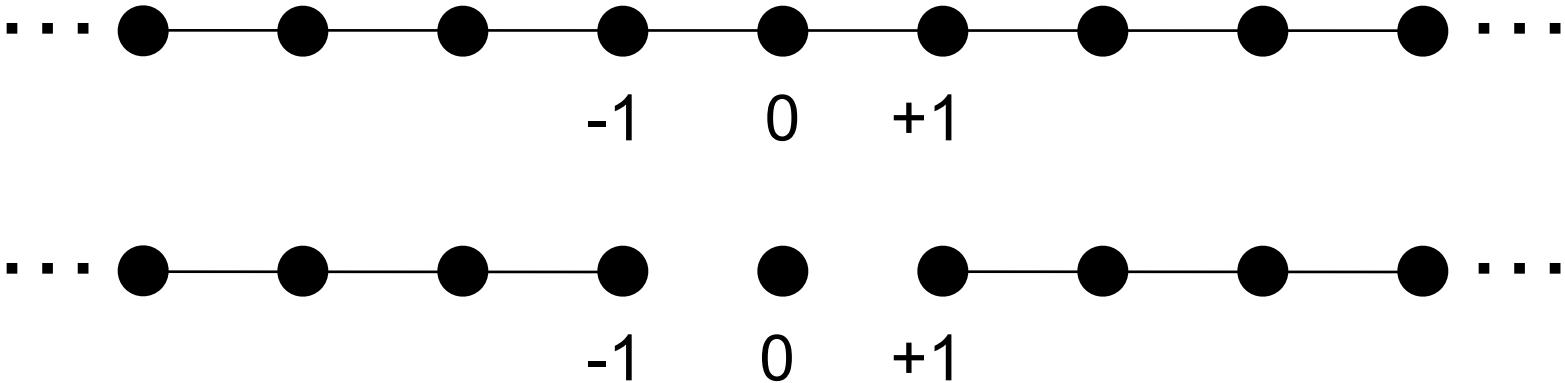
$$\langle j|\psi_q\rangle = \frac{1}{\sqrt{2}} [\langle j|q\rangle - \langle j|-q\rangle]$$

Bearing in mind that $\langle j|q\rangle = \frac{e^{iqjd}}{\sqrt{N}}$

$$\langle j|\psi_q\rangle = \frac{1}{\sqrt{2N}} [e^{iqjd} - e^{-iqjd}]$$

$$\langle j|\psi_q\rangle = i\sqrt{\frac{2}{N}} \sin(qjd)$$

$$\langle j | \psi_q \rangle = i \sqrt{\frac{2}{N}} \sin(qjd)$$



$$\hat{S}(E) = \sum_q \frac{|\psi_q\rangle\langle\psi_q|}{E - \varepsilon(q)}$$

$$\langle j | \hat{S}(E) | \ell \rangle = S_{j,\ell}(E) = \sum_q \frac{\langle j | \psi_q \rangle \langle \psi_q | \ell \rangle}{E - \varepsilon(q)}$$

$$S_{j,\ell}(E) = \frac{1}{N} \sum_q \frac{4 \sin(qjd) \sin(q\ell d)}{E - \varepsilon(q)}$$

$$S_{j,\ell}(E) = \frac{d}{2\pi} \int_{-\frac{\pi}{d}}^{\frac{\pi}{d}} dq \frac{4 \sin(qjd) \sin(q\ell d)}{E - \varepsilon(q)}$$

$$S_{1,1}(E) = \frac{d}{2\pi} \int_{-\frac{\pi}{d}}^{\frac{\pi}{d}} dq \frac{4 \sin^2(qd)}{E - \varepsilon(q)}$$

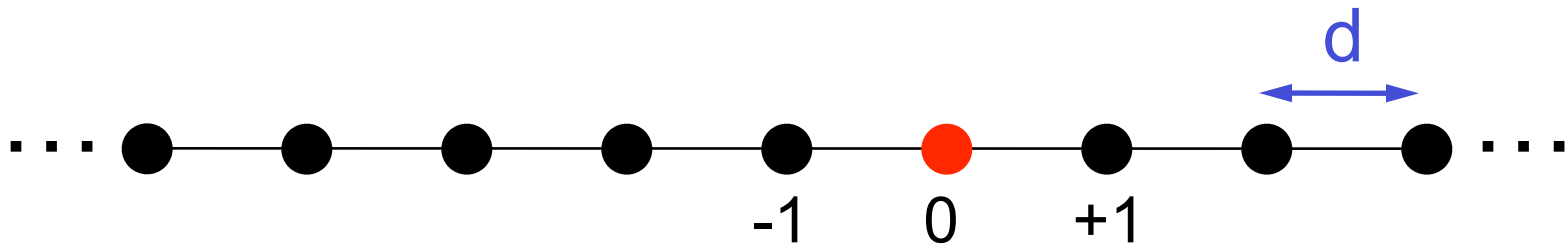
$$S_{1,1}(E) = \frac{-d}{2\pi} \int_{-\frac{\pi}{d}}^{\frac{\pi}{d}} dq \frac{\left[e^{2iqd} + e^{-2iqd} - 2 \right]}{E - \varepsilon(q)}$$

$$S_{1,1}(E) = \frac{-d}{2\pi} \int_{-\frac{\pi}{d}}^{\frac{\pi}{d}} dq \frac{\left[e^{2iqd} + e^{-2iqd} - 2 \right]}{E - \varepsilon(q)} =$$

$$\frac{-d}{2\pi} \left\{ \underbrace{\int_{-\frac{\pi}{d}}^{\frac{\pi}{d}} dq \frac{e^{2iqd}}{E - \varepsilon(q)}}_{\text{U.H.P}} + \underbrace{\int_{-\frac{\pi}{d}}^{\frac{\pi}{d}} dq \frac{e^{-2iqd}}{E - \varepsilon(q)}}_{\text{L.H.P}} - \underbrace{\int_{-\frac{\pi}{d}}^{\frac{\pi}{d}} dq \frac{2}{E - \varepsilon(q)}}_{\text{Either}} \right\}$$

$$S_{1,1}(E) = \frac{(E - \varepsilon_0) \pm \sqrt{(E - \varepsilon_0)^2 - 4t^2}}{2t^2}$$

Substitutional impurity



$$\hat{H} = \hat{H}_0 + \hat{V} \quad ; \quad \hat{V} = |0\rangle\lambda\langle 0|$$

$$\hat{G} = \hat{g} + \hat{g}\hat{V}\hat{G}$$

$$G_{j,l} = g_{j,l} + g_{j,0} T_{0,0} g_{0,l} \quad ; \quad T_{0,0} = \lambda [1 - g_{0,0} \lambda]^{-1}$$

What happens in the limit $\lambda \rightarrow \infty$?

$$\lim_{\lambda \rightarrow \infty} T_{0,0} = -\frac{1}{g_{0,0}} \quad \Rightarrow \quad G_{j,l} = g_{j,l} - g_{j,0} \left(\frac{1}{g_{0,0}} \right) g_{0,l}$$

Let's look at the element $G_{1,1}$:

$$G_{1,1} = g_{1,1} - g_{1,0} \left(\frac{1}{g_{0,0}} \right) g_{0,1} \quad g_{0,0} = g_{1,1}$$

$$G_{1,1} = \frac{g_{0,0}^2 - g_{1,0}g_{0,1}}{g_{0,0}}$$

We have shown that $g_{j,\ell}(E) = \left(\frac{i}{2t} \right) \frac{e^{iq_p(j-\ell)d}}{\sin(q_p d)}$

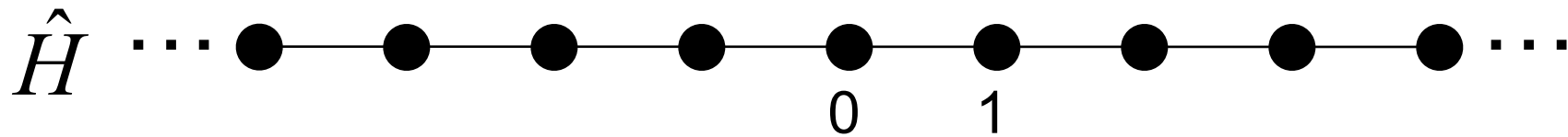
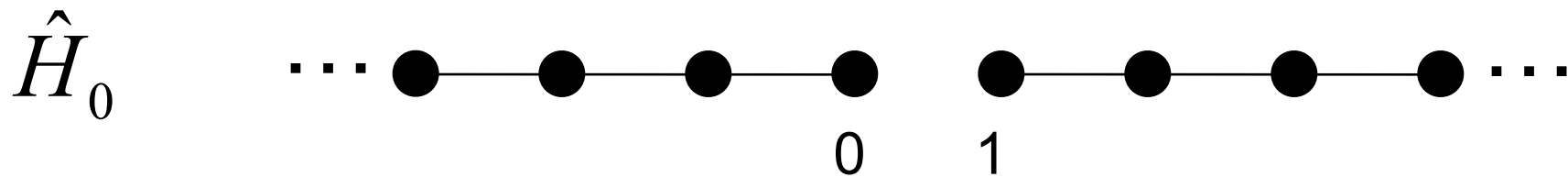
where $\cos(q_p d) = \left(\frac{E - \epsilon_0}{2t} \right)$

$$G_{1,1} = \left(\frac{i}{2t} \right) \frac{1 - e^{2iq_p d}}{\sin(q_p d)} \quad \Rightarrow \quad G_{1,1} = \frac{e^{iq_p d}}{t}$$

$$G_{1,1} = \frac{\cos(q_p d) + i \sin(q_p d)}{t}$$

$$G_{1,1} = \frac{(E - \varepsilon_0) \pm \sqrt{(E - \varepsilon_0)^2 - 4t^2}}{2t^2} = \mathcal{S}_{1,1}$$

We can simulate the presence of a wall by raising the on-site potential of a substitutional impurity to infinity ($\lambda \rightarrow \infty$). **But there are other ways...**



$$\hat{V} \text{ ---}$$

$$\hat{H} = \hat{H}_0 + \hat{V} \quad ; \quad \hat{V} = |0\rangle t \langle 1| + |1\rangle t \langle 0|$$

$$\hat{g} = \hat{S} + \hat{S} \hat{V} \hat{g}$$

$$g_{0,0} = S_{0,0} + S_{0,0} t g_{1,0} + \cancel{S_{0,1} t g_{0,0}}$$

$$g_{1,0} = \cancel{S_{1,0}} + \cancel{S_{1,0} t g_{1,0}} + S_{1,1} t g_{0,0}$$

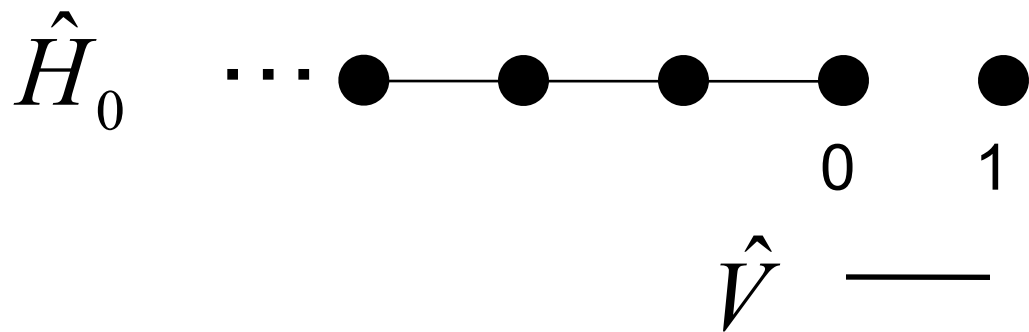
$$g_{0,0} = \left[1 - S_{0,0} t S_{1,1} t \right]^1 S_{0,0} \quad (S_{0,0} = S_{1,1})$$

$$g_{0,0} = \frac{S_{0,0}}{1 - S_{0,0}^2 t^2} \Rightarrow t^2 g_{0,0} S_{0,0}^2 + S_{0,0} - g_{0,0} = 0$$

$$S_{0,0} = \frac{-1 \pm \sqrt{1 + 4t^2 g_{0,0}^2}}{2t^2 g_{0,0}}$$

As expected

$$S_{0,0} = \frac{(E - \varepsilon_0) \pm \sqrt{(E - \varepsilon_0)^2 - 4t^2}}{2t^2}$$



$$\hat{H} = \hat{H}_0 + \hat{V} \quad ; \quad \hat{V} = |0\rangle t \langle 1| + |1\rangle t \langle 0|$$

$$\hat{G} = \hat{g} + \hat{g} \hat{V} \hat{G}$$



$$G_{1,1} = g_{1,1} + g_{1,0} t G_{1,1} + g_{1,1} t G_{0,1}$$

$$G_{0,1} = g_{0,1} + g_{0,0} t G_{1,1} + g_{0,1} t G_{0,1}$$

$$G_{1,1} = \left[\left(g_{1,1} \right)^{-1} - t g_{0,0} t \right]^{-1} \Rightarrow G_{1,1} = \left[\left(E - \varepsilon_0 \right) - t g_{0,0} t \right]^{-1}$$

Recursively, we have

$$G_{j+1,j+1} = \left[(E - \varepsilon_0) - t g_{j,j} t \right]^{-1}$$

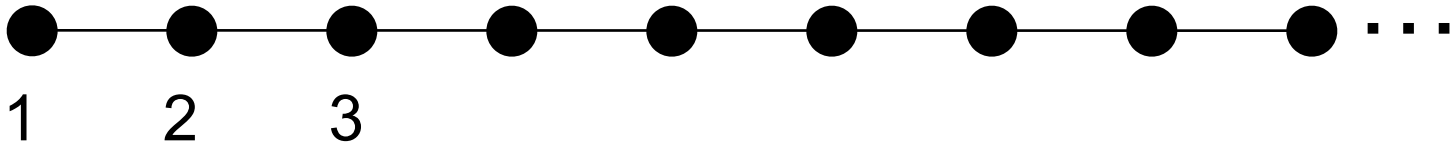
 **output**  **input**

In the limit of an infinitely long chain, output = input

$$S = \left[(E - \varepsilon_0) - t S t \right]^{-1} \Rightarrow t^2 S^2 - (E - \varepsilon_0) S + 1 = 0$$

Once again

$$S = \frac{(E - \varepsilon_0) \pm \sqrt{(E - \varepsilon_0)^2 - 4t^2}}{2t^2}$$

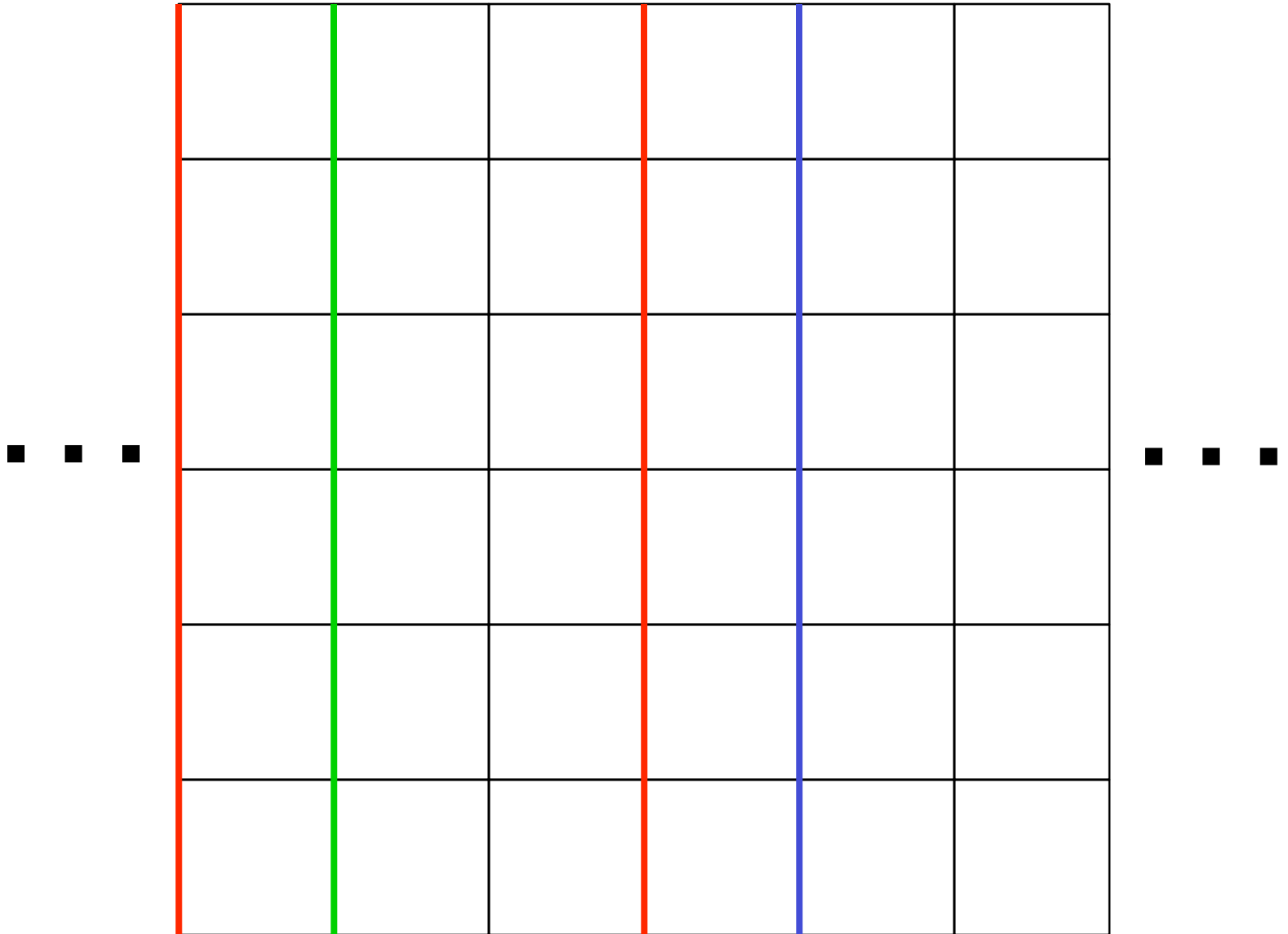


$$S = \frac{(E - \varepsilon_0) \pm \sqrt{(E - \varepsilon_0)^2 - 4t^2}}{2t^2}$$

Surface Green function

- Linear combination: $|\psi_q\rangle \equiv \frac{1}{\sqrt{2}} [|q\rangle - | -q\rangle]$
- Substitutional impurity ($\lambda \rightarrow \infty$)
- Connecting two semi-spaces
- Recursive method (asymptotic solution)

What if we have a layered geometry ?



Mixed representation

$$\hat{H} = \sum_{j,\ell} |j,\ell\rangle \varepsilon_0 \langle j,\ell| + \sum_{j,\ell,j',\ell'} |j,\ell\rangle t \langle j',\ell'|$$

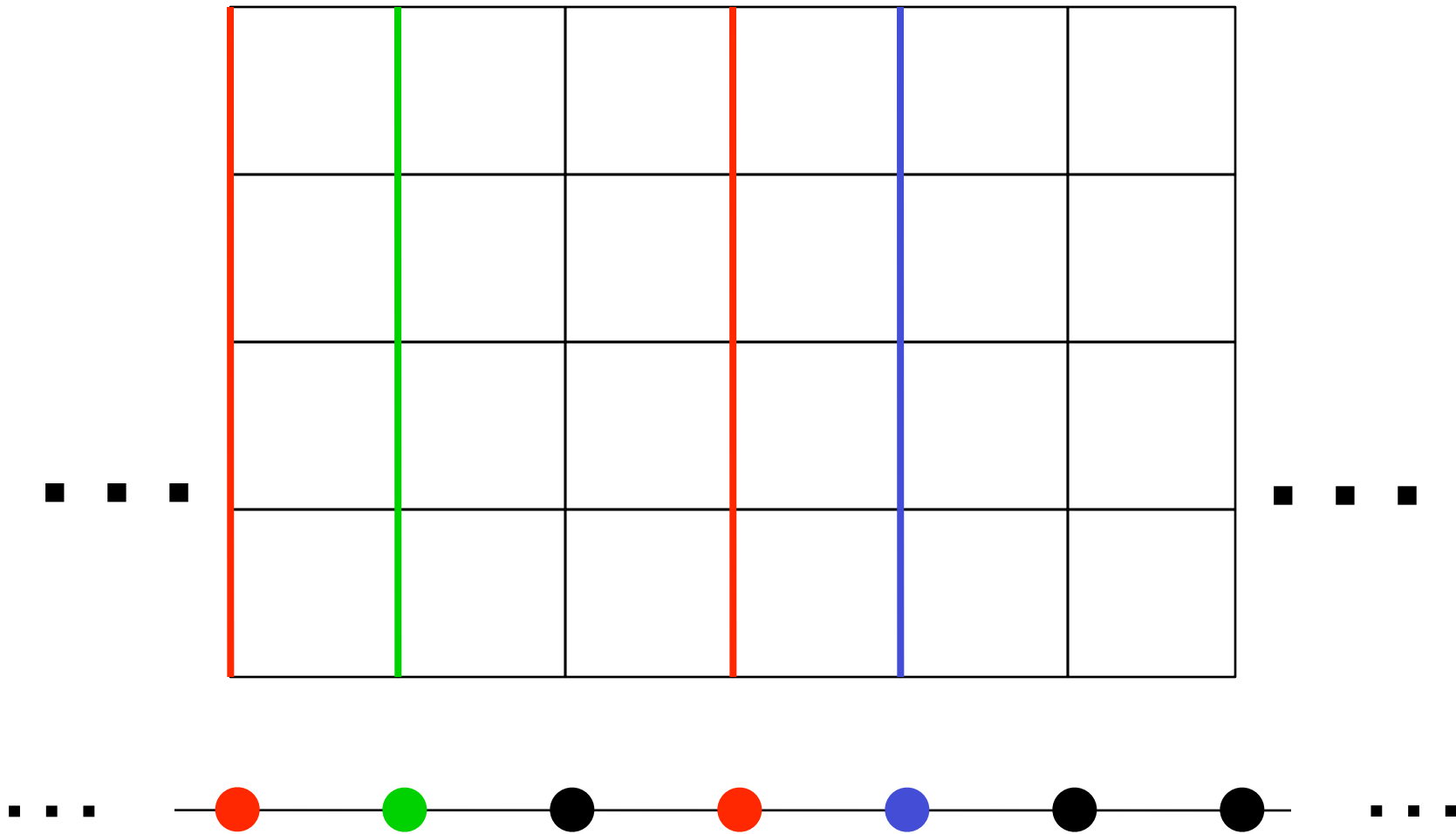
$$|j, q_y\rangle = \frac{1}{\sqrt{N}} \sum_{\ell} e^{iq_y \ell d} |j, \ell\rangle$$

$$|j, \ell\rangle = \frac{1}{\sqrt{N}} \sum_{q_y} e^{-iq_y \ell d} |j, q_y\rangle$$

$$\tilde{\varepsilon}(q_y) = \varepsilon_0 + 2t \cos(q_y d)$$

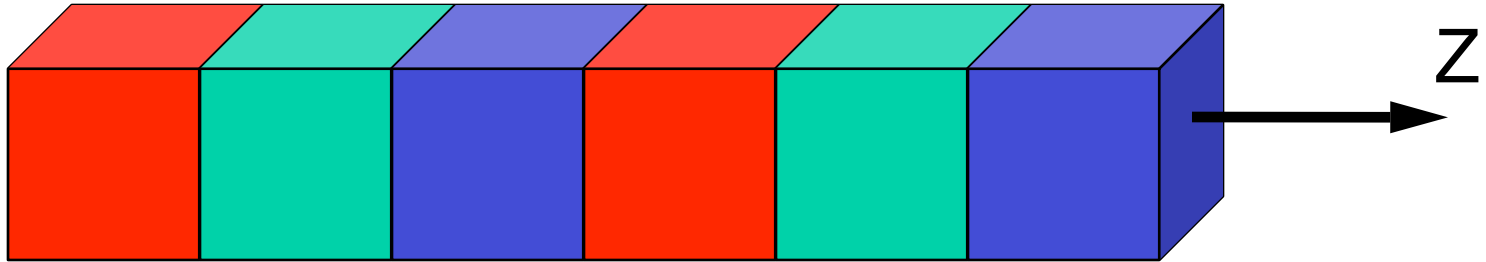

$$\hat{H} = \sum_{q_y} \left\{ \sum_j |j, q_y\rangle \tilde{\varepsilon}(q_y) \langle j, q_y| + \sum_j |j, q_y\rangle t \langle j \pm 1, q_y| \right\}$$

1D tight-binding Hamiltonian
with q_y -dependent parameters



2D square lattice is mapped into a collection of 1D chains

For a multi-layered material

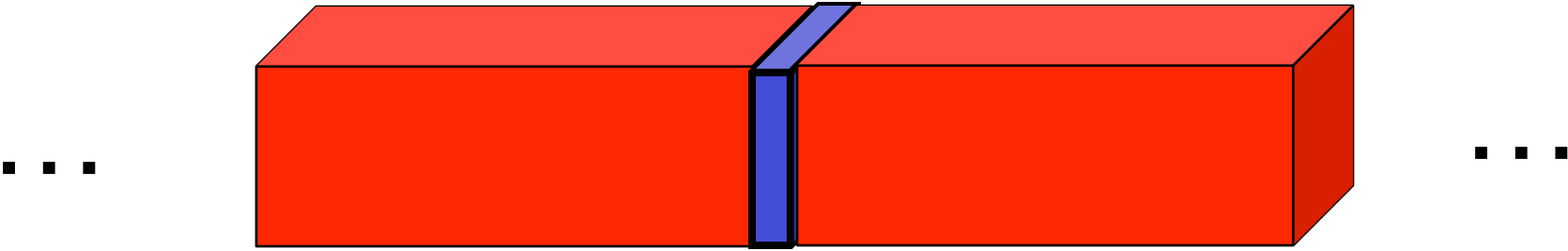
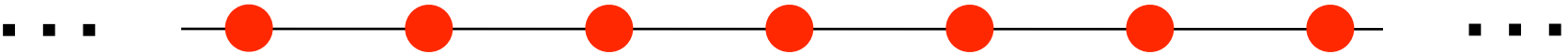


The Hamiltonian looks like

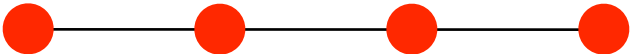
$$\hat{H} = \sum_{q_x, q_y} \left\{ \sum_j |j, q_x, q_y\rangle \tilde{\epsilon}(q_x, q_y) \langle j, q_x, q_y| + \sum_j |j, q_x, q_y\rangle t(q_x, q_y) \langle j \pm 1, q_x, q_y| \right\}$$

1D tight-binding Hamiltonian with
 q_x - and q_y -dependent parameters

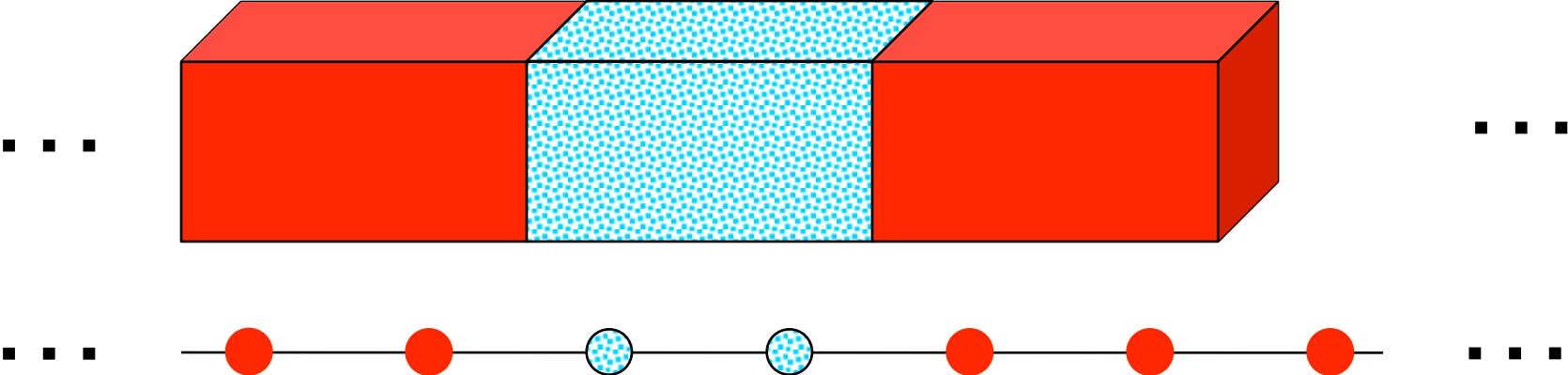
Out of our bag of tricks we can easily evaluate the GF for the following structures:



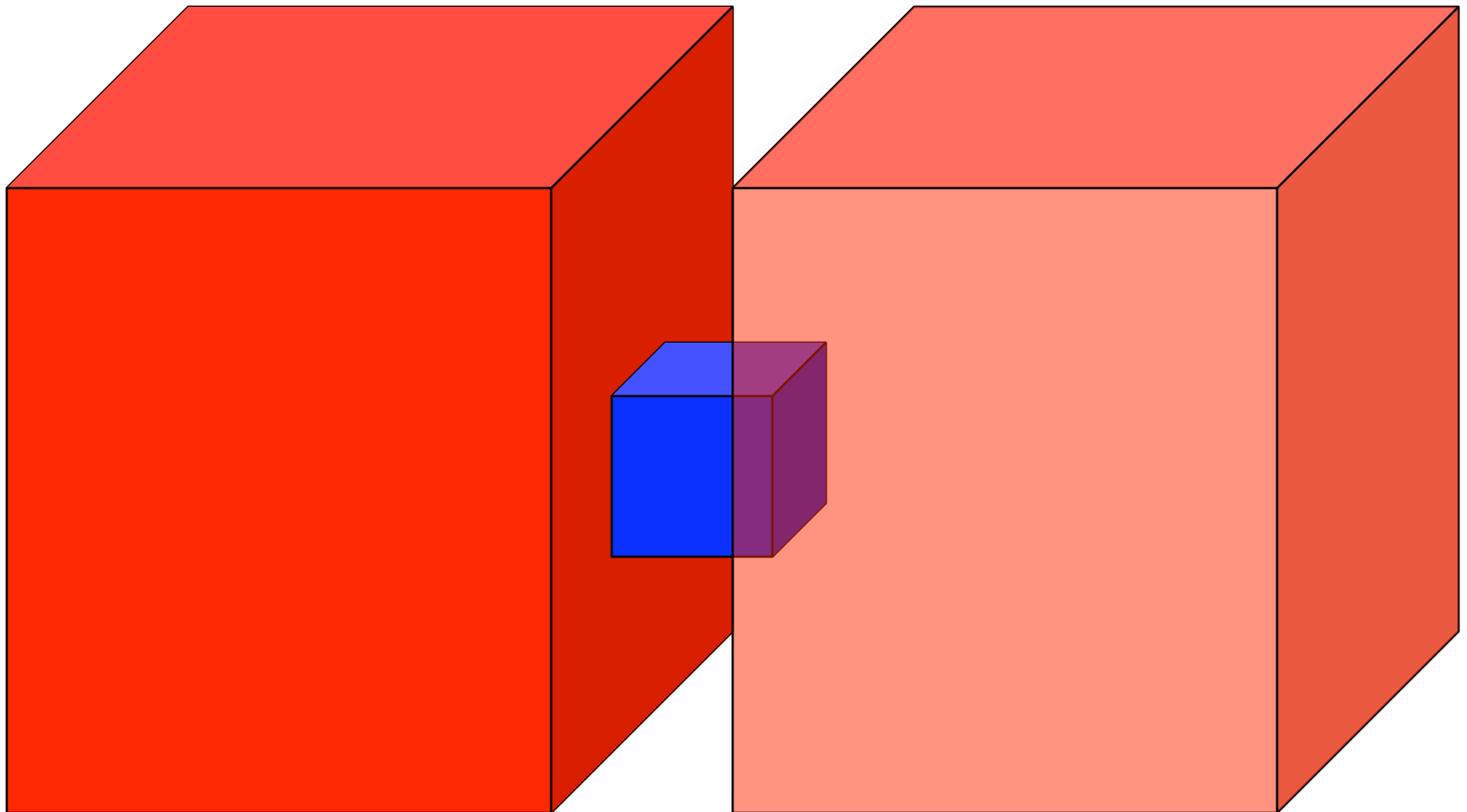
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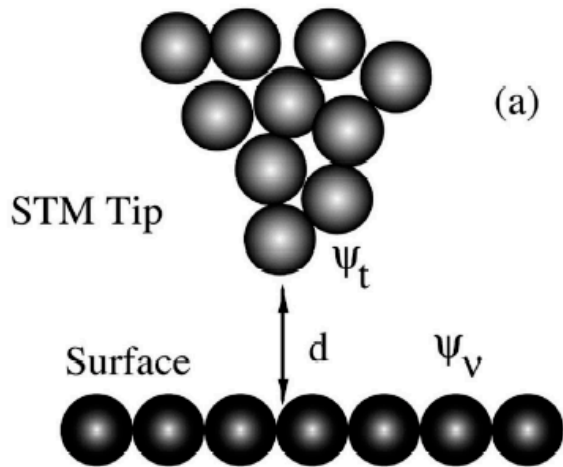


Out of our bag of tricks we can easily evaluate the GF for the following structures:



Measuring the LDOS (STM)

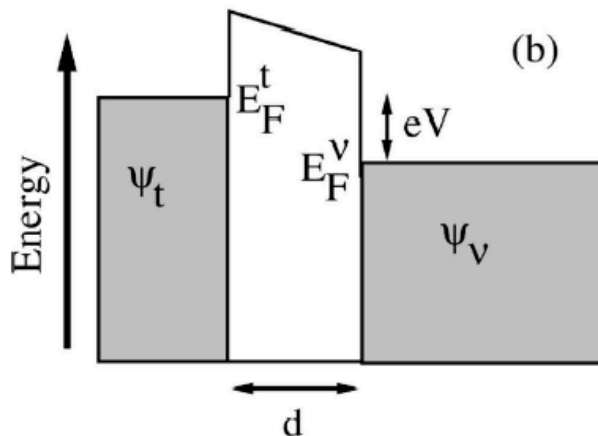
Tunneling Geometry



$$I(\vec{r}) \propto \int_0^{eV} dE \rho_t(E) LDOS(\vec{r}, E)$$

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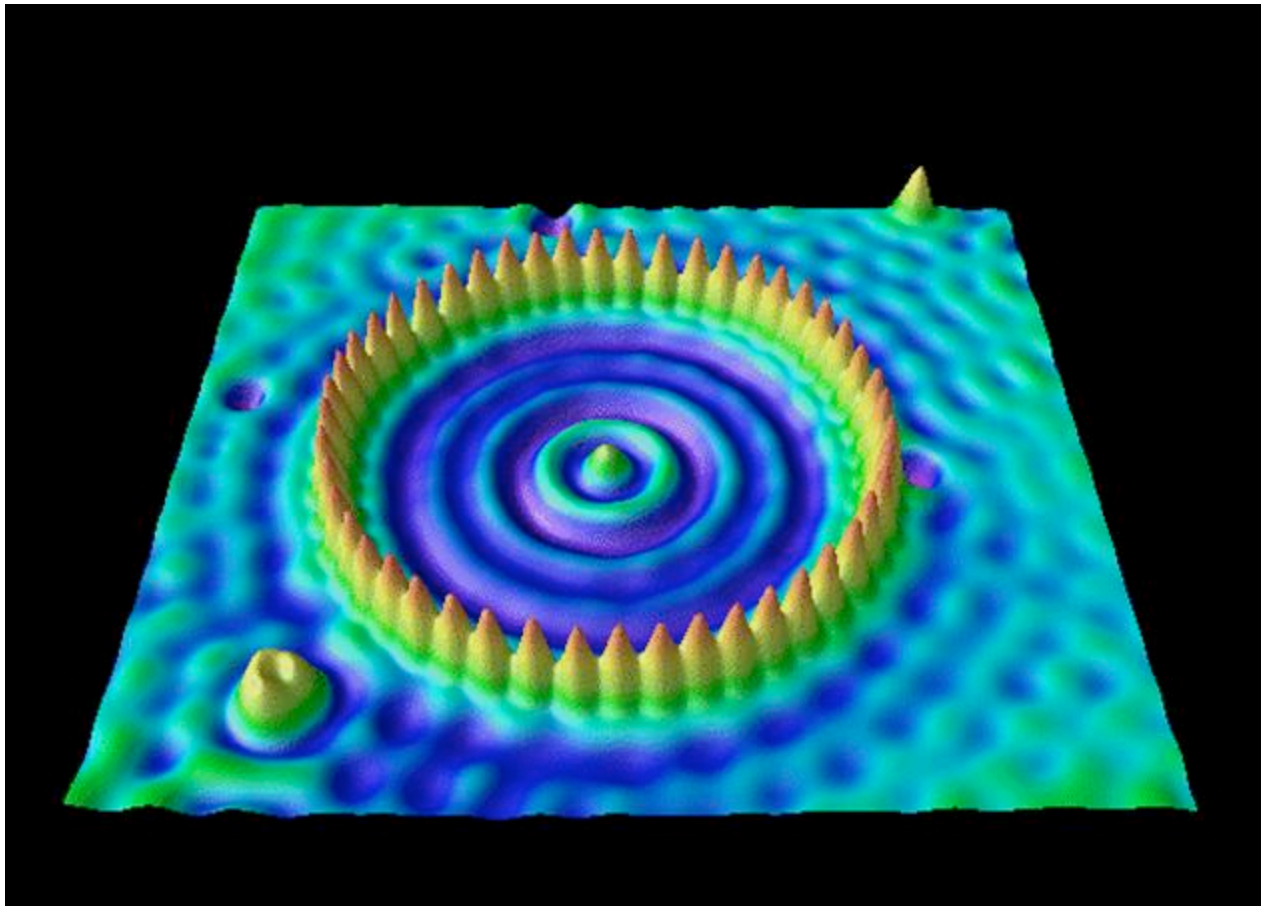
Corresponding Energy Diagram



$$\frac{dI}{dV}(\vec{r}, E) \propto LDOS(\vec{r}, E)$$

You might have seen this...

Quantum corral



Tutorial sheet

1) The bulk modulus κ of a solid is given by the second derivative of the total energy E_t with respect to the volume. Estimate the bulk moduli of alkali metals by assuming that the total energy is equivalent to the kinetic energy of the free-electron Fermi gas.

2) Consider the following tight-binding Hamiltonian for an infinite linear chain with nearest-neighbour hopping parameters t and on-site potentials ε_0 :

$$\hat{H} = \sum_j |j\rangle \varepsilon_0 \langle j| + \sum_j \left[|j\rangle t \langle j+1| + |j+1\rangle t \langle j| \right]$$

Show that the transformation $|q\rangle = \frac{1}{\sqrt{N}} \sum_j e^{iqjd} |j\rangle$

makes H diagonal

3) Consider the tight-binding Hamiltonian for a finite linear chain with N atoms that are a distance d apart. The nearest-neighbour hopping parameters and on-site potentials are t and ε_0 , respectively. Analogously to the semi-infinite case discussed in the lecture, find a suitable linear combination that diagonalizes this Hamiltonian. Discuss the energy quantization obtained.